Optimization of Non-Periodic Inspection of Structural Components by Bayesian Approach

Haoyu Huang, Jeremy Knopp, Manabu Tsunokai, Hiroo Asada

***Abstract*— This paper presents an advanced Bayesian analysis method to determine the appropriate non-periodic inspection intervals of fatigue-sensitive structures. The method is based on assumptions about the probability density functions of the time until crack initiation, law of crack propagation, probability of crack detection and failure rate before and after crack initiation. A major feature of this method is that even when there are uncertain parameters in the equations, the next inspection interval can be determined. Moreover, the probability density function of the uncertain parameters are updated according to the results of each inspection. A simulation study was conducted to evaluate the proposed method. The proposed method is evaluated statistically and it is shown that this Bayesian approach allows (a) evaluating accurately the inspection interval even with incorrect prior knowledge about the parameters and, (b) estimating accurately the reliability of a system, even when some of the parameters are uncertain.**

***Index Terms*— Bayesian Methods, Maintenance, Optimization, Probability Density Function, Reliability**

I. Introduction

There is a demand for performing maintenance of aging infrastructure more efficiently and effectively in many industries. For safety-critical infrastructure such as nuclear power plants, aircraft structural components, and aircraft turbine engines, the optimization of inspections with respect to cost and reliability is desired. Bayesian approaches have been introduced in [1] and [2] to determine the appropriate non-periodic inspection intervals for multiple components fatigue-sensitive structures so that, throughout the service life, the reliability remains above a predefined minimum level. As part of this approach, values of several uncertain parameters must be estimated. The method introduced in [3] is focused on aircraft structures and makes use of the appropriate equations for the law of crack propagation, the probability of crack detection and the failure rate after crack initiation.

In this paper, the Bayesian method of [3], which requires all the historical information about inspections, is modified so that only the information from the latest replacement or repair of components is required. This is achieved by introducing a conditional probability. In addition, the method is evaluated through a statistical analysis consisting in a large number of computational trials. This analysis was not possible at the time of the previous researches because of the lack of sufficient computational resources.

Manuscript received May 26th, 2017. This work was supported by the grant of the Asian Office of Aerospace Research and Development (AOARD).

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II. Non-Periodic Inspection Schedule Using Probabilistic Method

A. Basic Assumptions and Equations

A system is considered that consists in a specific number of elements. An element is such as it possesses only one fatigue-critical location. Throughout this paper, time is measured in the number of total accumulated cycles.

All elements are inspected at start-up and at the time of each scheduled inspection. Cracks and failures that do not cause the system to stop functioning, can only be detected during inspections. The following assumptions are made:

- The probability of detection of a crack is a function of the crack length.
- The probability of detection of an element failure is equal to one.
- When detected, all cracks or failures cause the concerned element to be repaired or replaced.
- After repair or replacement, an element regains its initial strength (same as a new one).
- No stress redistribution is considered after the occurrence of crack or failure.

1) Fatigue crack initiation

The time to crack initiation (TTCI), denoted by t and measured in number of cycles, is assumed to be a random variable with a probability density function that follows a Weibull distribution:

$$f_c(t|\beta) = \frac{\alpha}{\beta} \cdot \left(\frac{t}{\beta}\right)^{\alpha-1} \cdot exp\left[-\left(\frac{t}{\beta}\right)^{\alpha}\right] \qquad t > 0. \tag{1}$$

Additional uncertainty is introduced in the TTCI by the scale parameter β which is considered to be a random variable and is the first uncertain parameter of this study. The shape

parameter α is assumed to be deterministic for the sake of simplicity. The conditional distribution function of the TTCI is expressed by

$$F_c(t|\beta) = 1 - exp\left[-\left(\frac{t}{\beta}\right)^{\alpha}\right] \qquad t > 0. \tag{2}$$

2) Fatigue crack propagation

Fracture mechanics theory is used to determine the length of a propagating crack under random stress. For the purpose of this study, it is assumed that a crack grows according to the following law [4].

$$\frac{\mathrm{d}a}{\mathrm{d}t} = c \cdot a^{b/2}\,, \tag{3}$$

where a is the crack length, c and b are constants. Integrating (3) from the initial crack length a_0 at time of crack initiation t_c, to the current crack length a at time t, (4) is obtained.

$$a(t - t_c|c) = \left[-b'c(t - t_c) + {a_0}^{-b'}\right]^{-1/b'} \quad \text{where } b' = \frac{b-2}{2} \tag{4}$$

Uncertainty in the fatigue crack propagation is introduced by parameter c which is considered to be a random variable and is the second uncertain parameter of this study.

3) Probability of detection

The probability of detecting an existing crack of length a during an inspection is

$$D(a|d) = 1 - exp\left[-\left(\frac{a - a_{min}}{d - a_{min}}\right)^{\theta}\right]. \tag{5}$$

a_{min} denotes the minimum detectable length and θ is a constant. Uncertainty in the probability of crack detection is introduced by parameter d which is considered to be a random variable and is the third uncertain parameter of this study.

4) Failure rate and probability of safety

Failure of an element occurs when the random stress exceeds the strength of the element for the first time. An element can fail either before or after crack initiation. According to [4], the failure rate before and after crack initiation at time instant t_c is

$$h(t) = \begin{cases} exp(r) & \text{for } t \le t_c \\ \dfrac{\alpha_r}{\beta_r} \cdot \left(\dfrac{t}{\beta_r}\right)^{\alpha_r - 1} + exp(r) & \text{for } t > t_c. \end{cases} \tag{6}$$

For the sake of simplicity, parameter r, α_r, β_r, are assumed to be deterministic constants. The probability of safety U of an element before crack initiation and during operation from the start-up time T_l to time t is

$$U(t - T_l) = exp\left\{-\int_{T_l}^{t} h(\tau)\mathrm{d}\tau\right\} = exp\{-(t - T_l)\cdot exp(r)\} \quad \text{for } t \le t_c\,. \tag{7}$$

The probability of safety V of an element after crack initiation and during operation from the time of crack initiation t_c to time t is

$$V(t - t_c) = exp\left\{-\int_{t_c}^{t} h(\tau)\mathrm{d}\tau\right\} = exp\left\{-\frac{1}{{\beta_r}^{\alpha_r}}({t}^{\alpha_r} - {t_c}^{\alpha_r}) - (t - t_c)\cdot exp(r)\right\} \quad \text{for } t > t_c\,. \tag{8}$$

It should be noted that the functional equations used for fatigue crack initiation, crack propagation, probability of detection, failure rate and probability of safety are selected mainly to demonstrate the capabilities of our approach. It is a straightforward task to change any equations as well as their parameters depending on the case under consideration.

B. Possible Events and Probabilities

During the j-th inspection, performed at time T_j, all possible events and their probability must be considered in order to estimate the reliability of a certain element.

1) Events and probabilities that a failure is found

The event $\{A\text{: } j, l\}$ represents the event that an element is found to have failed at the time of the j-th inspection T_j (or during the time interval $[T_{j-1}, T_j]$). This event is one of the two mutually exclusive events $E_{1,j}$ or $E_{2,j}$.

The event $E_{1,j}$ represents that the element failed before crack initiation, sometime between the two previous inspections, during the time interval $[T_{j-1}, T_j]$. $E_{1,j}$ consists in two sub-events: (a) no crack would have initiated in the element before T_j if failure did not occur sometime during the time interval $[T_{j-1}, T_j]$ and, (b) a crack would have initiated in the element at time instant t ($T_{j-1} < t < T_j$) if failure did not occur during the time interval $[T_{j-1}, t]$. The probability $P_{1,j}$ of event $E_{1,j}$ is given by the sum of the probabilities of these two sub-events.

$$P_{1,j} = \{1 - F_c(T_j - T_l|\beta)\}\cdot\{U(T_{j-1} - T_l) - U(T_j - T_l)\} + \int_{T_{j-1}}^{T_j} f_c(t - T_l|\beta)\cdot\{U(T_{j-1} - T_l) - U(t - T_l)\}\mathrm{d}t \tag{9}$$

The event $E_{2,j}$ represents that the element failed after crack initiation, sometime between the two previous inspections, during the time interval $[T_{j-1}, T_j]$. Event $E_{2,j}$ also consists in two sub-events (a) and (b).

(a) A crack initiated at time instant t in the time interval $[T_i, T_{i+1}]$ where $i = l, \dots, j\text{-}2$. The crack was not detected during all subsequent inspections (from inspection T_{i+1} to inspection T_{j-1}).

(b) A crack initiated at time instant t in the time interval $[T_{j-1}, T_j]$ and the element failed sometime during the time interval $[t, T_j]$.

The probability $P_{2,j}$ of event $E_{2,j}$ is given by the sum of the probabilities of the two sub-events.

$$P_{2,j} = \sum_{i=l}^{j-2} \left\{ \begin{array}{c} f_c(t-T_l|\beta)\cdot U(t-T_l)\cdot \\ \left[V(T_{j-1}-t)-V(T_j-t)\right]\cdot \\ \int_{T_i}^{T_{i+1}} \left[\prod_{k=i+1}^{j-1}\{1-D(a(T_k-t|c)|d)\}\right]\mathrm{d}t \end{array} \right\} + \int_{T_{j-1}}^{T_j} f_c(t-T_l|\beta)\cdot U(t-T_l)\cdot\{1-V(T_j-t)\}\mathrm{d}t \quad (10)$$

2) Events and probabilities that a crack is found

The event $\{B(a_j)\colon j, l\}$ represents the event that an element is found not to have failed at the time of the j-th inspection T_j and that a crack of length between a_j and $a_j + \mathrm{d}a_j$ is detected. This event consists of only one event $E_{3,j}$. Since a crack of length between a_j and $a_j + \mathrm{d}a_j$ is detected during the j-th inspection, the time of initiation of the crack t_c can be computed from (4) as follows:

$$t_c = T_j + \frac{1}{b'c}\left({a_j}^{-b'} - {a_0}^{-b'}\right). \quad (11)$$

The differential $\mathrm{d}t_c$ is:

$$\mathrm{d}t_c = \left|\frac{\mathrm{d}t_c}{\mathrm{d}a_j}\right|\mathrm{d}a_j = \frac{\mathrm{d}a_j}{c\cdot {a_j}^{b/2}}. \quad (12)$$

The probability $p_{3,j}(a_j)da_j$ of event $E_{3,j}$ is

$$p_{3,j}(a_j)\mathrm{d}a_j = f_c(t_c-T_l|\beta)\mathrm{d}t_c\cdot U(t_c-T_l)\cdot V(T_j-t_c)\cdot\left[\prod_{k=l+1}^{j-1}\{1-\delta\cdot D(a(T_k-t_c|c)|d)\}\right]\cdot D(a_j|d), \quad (13)$$

where δ is

$$\delta = \begin{cases} 1 & \text{for } T_k > t_c \\ 0 & \text{for } T_k < t_c \end{cases}. \quad (14)$$

By replacing t_c and $\mathrm{d}t_c$ in (13) by their expressions defined in (11) and (12), the probability $p_{3,j}(a_j)da_j$ can be expressed as a function of a_j only.

3) Events and probabilities where nothing is found

The event $\{C\colon j, l\}$ represents the event that the element is found not to have failed or cracked at the time of the j-th inspection T_j. This event is one of the two mutually exclusive events $E_{4,j}$ or $E_{5,j}$.

The event $E_{4,j}$ represents that the element did not fail during the time interval $[T_{j-1}, T_j]$ and that no crack is present. The probability $P_{4,j}$ of event $E_{4,j}$ is

$$P_{4,j} = \left\{1-F_c(T_j-T_l|\beta)\right\}\cdot U(T_j-T_l). \quad (15)$$

The event $E_{5,j}$ represents that the element did not fail during the time interval $[T_{j-1}, T_j]$ but that a crack initiated at time instant t in the time interval $[T_i, T_{i+1}]$ where $i = l, \ldots, j\text{-}2$, but was not detected during all subsequent inspections (from inspection T_{i+1} to inspection T_j). The probability $P_{5,j}$ of event $E_{5,j}$ is

$$P_{5,j} = \sum_{i=l}^{j-1}\left\{\int_{T_i}^{T_{i+1}} f_c(t-T_l|\beta)\cdot U(t-T_l)\cdot V(T_j-t)\cdot\left[\prod_{k=i+1}^{j}\{1-D(a(T_k-t|c)|d)\}\right]\mathrm{d}t\right\}. \quad (16)$$

4) Probabilities of each event

The probabilities of the events $\{A\colon j, l\}$, $\{B(a_j)\colon j, l\}$ and $\{C\colon j, l\}$ are

$$P\{A\colon j,l\} = P_{1j,} + P_{2,j}, \quad (17)$$

$$P\{B(a_j)\colon j,l\} = p_{3,j}(a_j)\mathrm{d}a_j, \quad (18)$$

$$P\{C\colon j,l\} = P_{4,j} + P_{5,j}. \quad (19)$$

III. Bayesian Method for Uncertain Parameters

The parameters that appear in the equations related to fatigue cracks, (2), (4) and (5), are uncertain parameters that depend on environmental conditions. In order to solve the problem of these uncertainties, a Bayesian method is applied. These parameters are modified according to the information from the inspections results.

A. Prior Joint Density Function

The three parameters β, c and d are considered as possible sources of uncertainty. Initially, it is assumed they are jointly and uniformly distributed according to the following prior density function:

$$f^0(\beta,c,d) = \frac{1}{(\beta_{max}-\beta_{min})(c_{max}-c_{min})(d_{max}-d_{min})} = \text{constant} = f^0, \quad (20)$$

where

$$\beta_{min} \le \beta \le \beta_{max};\ \ c_{min} \le c \le c_{max};\ \ d_{min} \le d \le d_{max}. \quad (21)$$

B. Conditional Probabilities

In the method introduced in [3], the likelihood function of the possible events concerning an element at the time of inspection, is determined from the historical information of all previous inspections since start-up. This determination is not possible if this information is incomplete. Using conditional probabilities, the Bayesian analysis method can solve this problem. However, the information from the time of the last repair or replacement of an element is still necessary.

1) Conditional probabilities after repair during last inspection

A^Y represents the event that a failure was found during the j-th inspection. The probability of event A^Y can be derived from (17) and is

$$P\{A^Y: j|Rep^{j-1}\} = \{1 - F_c(T_j - T_{j-1}|\beta)\} \cdot \{1 - U(T_j - T_{j-1})\} + \int_{T_{j-1}}^{T_j} f_c(t - T_{j-1}|\beta) \cdot \{1 - U(t - T_{j-1})\}\mathrm{d}t + \int_{T_{j-1}}^{T_j} f_c(t - T_{j-1}|\beta) \cdot U(t - T_{j-1}) \cdot \{1 - V(T_j - t)\}\mathrm{d}t\,, \quad (22)$$

where Rep^{j-1} represents the event that the element was repaired during the *(j-1)*-th inspection.

B^Y represents the event that a crack was found during the *j*-th inspection. The probability of event B^Y can be derived from (18) and is

$$P\{B^Y(a_j): j|\mathrm{Rep}^{j-1}\} = f_c(t_c - T_{j-1}|\beta)\left|\frac{\mathrm{d}t_c}{\mathrm{d}a_j}\right|\Delta a \cdot U(t_c - T_{j-1}) \cdot V(T_j - t_c) \cdot D(a_j|d). \quad (23)$$

C^Y represents the event that nothing was found during the *j*-th inspection. The probability of event C^Y can be derived from (19) and is

$$P\{C^Y: j|\mathrm{Rep}^{j-1}\} = \{1 - F_c(T_j - T_{j-1}|\beta)\} \cdot U(T_j - T_{j-1}) + \int_{T_{j-1}}^{T_j} f_c(t - T_{j-1}|\beta) \cdot U(t - T_{j-1}) \cdot V(T_j - t) \cdot \{1 - D(a(T_j - t|c)|d)\}\mathrm{d}t\,. \quad (24)$$

2) Conditional probabilities after no repair during last inspection

A^N represents the event that a failure was found during the *j*-th inspection. The probability of the event A^N is

$$P\{A^N: j, l|No^{j-1}, l\} = \frac{P\{A^N: j, l \cap \mathrm{No}^{j-1}, l\}}{P\{\mathrm{No}^{j-1}, l\}}, \quad (25)$$

where No^{j-1} represents the event that the element was not repaired during the *(j-1)*-th inspection, and *l* indicates that the last repair or replacement occurred at time T_l. The numerator $P\{A^N: j, l \cap \mathrm{No}^{j-1}, l\}$ in (25) has the same form as $P\{A: j, l\}$ in (17), but with the restriction that $l \neq (j-1)$. The denominator $P\{\mathrm{No}^{j-1}, l\}$ in (25) can be computed as $P\{C: j-1, l\}$ in (19).

B^N represents the event that a crack was found during the *j*-th inspection. The probability of the event B^N is

$$P\{B^N(a_j): j, l|\mathrm{No}^{j-1}, l\} = \frac{P\{B^N(a_j): j, l \cap \mathrm{No}^{j-1}, l\}}{P\{\mathrm{No}^{j-1}, l\}}. \quad (26)$$

The numerator $P\{B^N(a_j): j, l \cap \mathrm{No}^{j-1}, l\}$ in (26) has the same form as $P\{B(a_j): j, l\}$ in (18), but with the restriction that $l \neq (j-1)$.

C^N represents the event that nothing was found during the *j*-th inspection. The probability of the event C^N is

$$P\{C^N: j, l|\mathrm{No}^{j-1}, l\} = \frac{P\{C^N: j, l \cap \mathrm{No}^{j-1}, l\}}{P\{\mathrm{No}^{j-1}, l\}}. \quad (27)$$

The numerator $P\{C^N: j, l \cap \mathrm{No}^{j-1}, l\}$ in (27) has the same form as $P\{C: j, l\}$ in (19), but with the restriction that $l \neq (j-1)$.

3) Likelihood function

Only the information from the last repair or replacement is needed to determine the likelihood function with the Bayesian analysis method using conditional probabilities. The likelihood function for element *m* after the *j*-th inspection is

$$LF_j^{(m)} = P_m\{Y: j|\mathrm{Rep}^{j-1}\} \text{ or } P_m\{X: j, l|\mathrm{No}^{j-1}, l\}. \quad (28)$$

In (28), *Y* stands for either event A^Y, B^Y or C^Y, *X* stands for either A^N, B^N or C^N, depending on the result of the *j*-th inspection of element *m*. The probability $P_m\{*\}$ refers to the relevant probability in (22) to (27).

The likelihood function LF_j for the entire system after the *j*-th inspection is

$$LF_j = \prod_{m=1}^{M} LF_j^{(m)}\,, \quad (29)$$

where *M* is the total number of elements in the system.

4) Posterior probability density function

The posterior joint probability density function of parameters β, *c* and *d* after the *j*-th inspection is

$$f^j(\beta, c, d) = \frac{LF_j \cdot f^{j-1}(\beta, c, d)}{\int_{\beta_{min}}^{\beta_{max}} \int_{c_{min}}^{c_{max}} \int_{d_{min}}^{d_{max}} S\mathrm{d}\beta\mathrm{d}c\mathrm{d}d}, \quad (30)$$

where *S* is the sum of the probabilities of all the possible events at the time of the *j*-th inspection.

IV. Reliability Computation

Inspection intervals are determined according to the computed reliability of the entire system. The reliability of the elements of the system at the instant t^* in the time interval $[T_j, T_{j+1}]$, where T_j is the time of the *j*-th inspection, depends on whether the elements have been repaired or replaced during the *j*-th inspection.

A. Reliability of Elements Repaired or Replaced During the j-th Inspection

When an element is repaired or replaced during the *j*-th inspection, the condition of the element is reset and the start-up time T_l is equal to T_j. The reliability $R(t^*: \mathrm{Rep})$ of an element, in other words, the probability of an element survival, is computed as the sum of the following two probabilities: (a) the probability that the element will survive during the time interval $[T_j, t^*]$ and that no crack will initiate before t^* and, (b) the probability that a crack will initiate sometime during the time interval $[T_j, t^*]$, but the element will survive this time interval.

Thus, the reliability $R(t^*: \mathrm{Rep})$ is

$$R(t^*: \mathrm{Rep}) = \{1 - F_c(t^* - T_j|\beta)\} \cdot U(t^* - T_j) + \int_{T_j}^{t^*} f_c(t - T_j|\beta) \cdot U(t - T_j) \cdot V(t^* - t)\mathrm{d}t\,. \quad (31)$$

B. *Reliability of Elements not Repaired nor Replaced During the j-th Inspection*

The reliability $R(t^*\text{: No})$ of an element is computed as the sum of three probabilities divided by the probability that nothing was found during the j-th inspection, $P\{C: j, l\}$. The three probabilities are (a) the probability that the element will survive the time interval $[T_l, t^*]$ and that no crack will initiate before t^*, (b) the probability that a crack will initiate sometime during the time interval $[T_j, t^*]$, but the element will survive the time interval$[T_l, t^*]$ and, (c) the probability that a crack initiated at the instant t during the time interval $[T_i, T_{i+1}]$ where $i = l, \dots, j\text{-}1$. This crack was not detected during all subsequent inspections (from inspection T_{i+1} to inspection T_j) and the element will survive the time interval $[T_l, t^*]$.

Thus, the reliability $R(t^*\text{: No})$ is

$$R(t^*\text{: No.}) = \frac{Z}{P\{C: j, l\}}, \tag{32}$$

where Z is

$$\begin{aligned} Z = {} & \{1 - F_c(t^* - T_l|\beta)\} \cdot U(t^* - T_l) \\ & + \int_{T_j}^{t^*} f_c(t - T_l|\beta) \cdot U(t - T_l) \cdot V(t^* - t)\mathrm{d}t \\ & + \sum_{i=l}^{j-1} \left\{ \int_{T_i}^{T_{i+1}} f_c(t - T_l|\beta) \cdot U(t - T_l) \cdot V(t^* - t) \right. \\ & \left. \cdot \left[\prod_{k=i+1}^{j} \{1 - D(a(T_k - t|c)|d)\} \right] \mathrm{d}t \right\}. \end{aligned} \tag{33}$$

C. *Determination of Reliability and Next Inspection Interval*

The reliability $\overline{\tilde{R}_M}(t^*)$ of the entire system at the instant t^* is determined by an integral over the whole variable space from the probability density function of the uncertain parameters β, c and d.

$$\overline{\tilde{R}_M}(t^*) = \int_{\beta_{min}}^{\beta_{max}} \int_{c_{min}}^{c_{max}} \int_{d_{min}}^{d_{max}} \tilde{R}_M(t^*|\beta, c, d) \cdot f^j(\beta, c, d)\mathrm{d}\beta\mathrm{d}c\mathrm{d}d\,, \tag{34}$$

where

$$\tilde{R}_M(t^*|\beta, c, d) = \left[\prod_{m=1}^{M_1} R_m(t^*\text{: Rep.}) \right] \cdot \left[\prod_{m=1}^{M_2} R_m(t^*\text{: No.}) \right]. \tag{35}$$

In (35), M_1 is the number of elements repaired or replaced during the j-th inspection, M_2 is the number of elements found intact after the j-th inspection, and M_1+M_2 = M is the total number of elements. R_m is the reliability of the m-th element and is defined in (31) and (32).

Assuming that the entire system must maintain its reliability above a specified design level R_{design} throughout its service life, the reliability at time t^* after an inspection performed at time T_j should satisfied (36).

$$\overline{\tilde{R}_M}(t^*) \geq R_{\text{design}} \tag{36}$$

In order to determine the time of the next inspection, a simulation is performed and t^* is increased by a time increment until (36) is not satisfied anymore. The maximum value of t^* that satisfies (36) is the time of the next inspection. The steps of the simulation to determine the time of the next inspection are the following.

(1) Simulation of the initiation and propagation of fatigue cracks and rate of failures, using real values for all the parameters.

(2) The reliability of the entire system is estimated without using the results of the first step. As the values of the uncertain parameters are not known, the posterior joint density function of these parameters at time T_j are used. The maximum value of t^* that satisfies equation (36) is determined, and the next inspection time T_{j+1} is set to this value.

(3) Virtual inspection of the system at time T_{j+1} is performed, accordingly to the results of the first step.

(4) Inspection results are used to compute the likelihood and posterior joint density functions at time T_{j+1} .

(5) Back to the first step to determine the time of the next inspection.

V. Numerical Results

A. *Value of the Parameters in Numerical Simulations*

The system considered in this study is assumed to have 50 elements (M=50). Its service life is limited to 30,000 cycles and the minimum reliability level for entire system is set to 0.8 (R_{design}=0.8). The values of all the parameters used in numerical simulations are shown in Table 1.

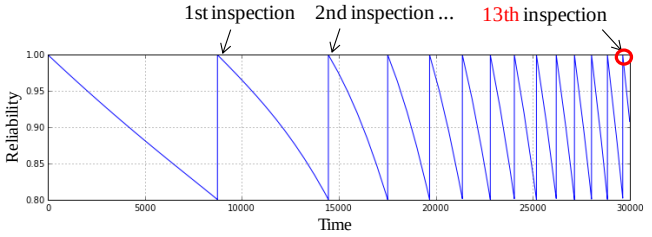
The true value of the parameters is always used when simulating the behavior of the elements of the system such as crack initiation, propagation, detection and rate of failure. However, when estimating the reliability, the parameters β, c, and d are either fixed to their true value, fixed to an incorrect value, or are uncertain (they have a probability density function).

TABLE 1
VALUES OF PARAMETERS IN NUMERICAL SIMULATIONS

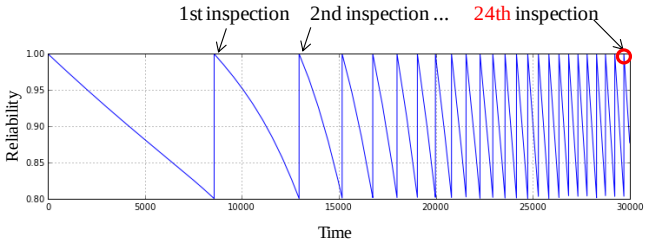
Item		True values	Range of uncertainty
General	Service life	30,000 cycles	-
	Minimum level of R_{design}	0.8	-
	Number of element M	50	-
Crack Initiation	Parameter α	4.0	-
	Parameter β	40,000 cycles	20,000 ~ 60,000
Crack Propagation	Parameter b	2.96	-
	Parameter c	$1.6*10^{-4}$mm$^{-0.48}$/cycle	$0.6*10^{-4}$ ~ $2.6*10^{-4}$
	Initial crack length a_0	2.5mm	-
Crack detection	Parameter $a_{\min}$	2.5mm	-
	Parameter θ	1.4	-
	Parameter d	40mm	20 ~ 60
Reliability	Parameter r	-14.5	-
	Parameter α_r	3.7	-
	Parameter β_r	8,000 cycles	-

B. *Results with Fixed Parameters*

First, as a demonstration, the results in the case of known parameters β, c and d are shown in Fig. 1. The results for two values of β are compared. The vertical axis represents the estimated reliability of the system and the horizontal axis the time. Each time an inspection is performed, the reliability increases to 1.0. This results show that reasonable inspection intervals are obtained with our method, that is, the required inspection intervals become shorter over time. According to (2), a smaller value of β means that a crack will initiate earlier and thus, more frequent inspections are required (see Fig. 1 (b)).



(a) β= 40,000

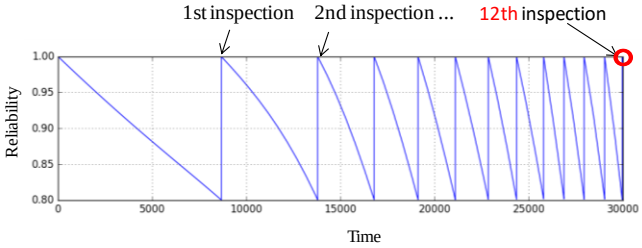


(b) β=30,000

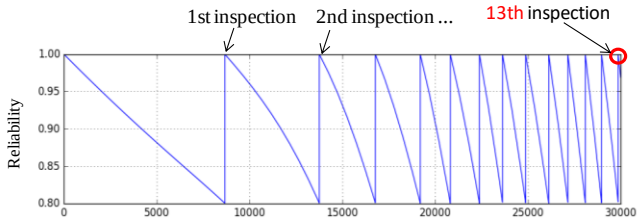
Fig. 1. Reliability of the system with fixed parameter.

C. *Results with Uncertain Parameters*

The results when parameters β, and either c or d are uncertain are shown in Fig. 2. In these cases, the prior probability density function of those parameters follows a uniform distribution, from the minimum to the maximum values specified in Table 1. Reliability is determined from the probability density functions of the parameters, which are updated according to the results of each inspection. Fig. 2 shows that similar inspection intervals for the case of known parameters value and β = 40,000 are obtained, even when two parameters are uncertain.



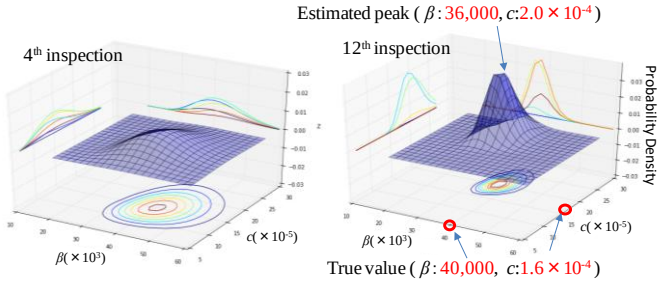
(a) β, c uncertain



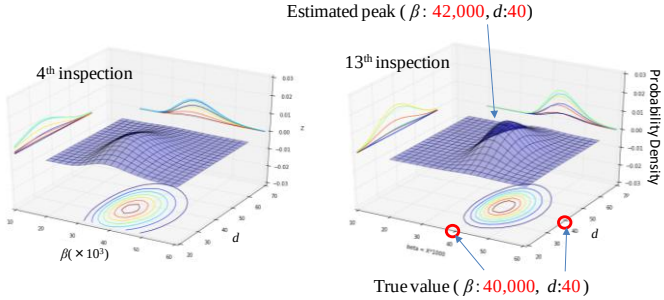
(b) β, d uncertain

Fig. 2. Reliability of the system with uncertain parameters.

Fig. 3 shows the probability density function of the uncertain parameters after a number of inspections. As the number of inspections performed increases, the peak near the true value becomes more prominent. It indicates that the values of the parameters are correctly estimated by the method.



(a) β, c uncertain



(b) β, d uncertain

Fig. 3. Probability density function of the parameters after a number of inspections.

VI. Statistical Analysis

The initiation time of a fatigue crack is considered to be a random variable that follows a Weibull distribution. Furthermore, the parameters β, c and d may be uncertain.

Because a simulation is a stochastic process, in order to evaluate our method, it is necessary to perform a statistical analysis. According to a preliminary examination, where different number of trials from 500 to 2000 were compared, performing 1000 or more trials is enough to obtain good statistical analysis results.

A. *Basic Statistical Properties*

In order to evaluate the Bayesian estimation of the uncertain parameters in several cases, statistical analysis is performed. The average, standard deviation and coefficient of variation values of the total number of inspections and of the peak values of the parameters after each trial are shown in

Table 2. The results are obtained from 2000 trials and the numbers in parentheses are the true values of the parameters. The last row shows the results when all parameters are fixed and no Bayesian estimation is performed.

The average peak value of parameter β is a good estimation of the true value. Although the average number of inspections when β is uncertain is close to the true value, the standard deviation is relatively large. This is because β has a significant impact on the determination of the inspection intervals. The average peak value of parameter c is 20% greater than the true value, leading to an underestimation of the number of inspections. Because the Bayesian estimation reaches a local minimum when parameter c is uncertain, more information is necessary to improve its estimation. The estimation of parameter d is not as good, notably because of a relatively large standard deviation. However, when the uncertain parameter is d, the estimation of the number of inspections is very good. It means that this parameter does not have a significant influence on the reliability.

TABLE 2
STATISTICAL RESULTS OF BAYESIAN ESTIMATION

Uncertain parameter	Number of inspections			Parameter		
	Average	SD	COV(%)	Average	SD	COV(%)
β	13.60	2.81	20.7	41,080 (40,000)	4,800	11.7
c	11.66	1.63	14.0	1.964E-04 (1.6E-04)	2.85E-05	14.5
d	12.79	0.49	3.8	38.92 (40)	11.18	28.7
True value	12.98	0.17	1.3	-	-	-

B. *Statistical Analysis of Several Prior Density Functions*

The inspection intervals estimated by the method depend on the prior density function of the uncertain parameters. Especially for the first inspections, when there is not enough information to estimate accurately the range of the parameters, the prior density function is the main factor influencing the length of the inspection intervals.

The results of the statistical analysis of several prior density functions of β are compared in

Table 3. The first three rows are the results when β is fixed and no Bayesian update is performed. As the true value of β is 40,000, the other values correspond to cases where one has an incorrect knowledge about the value of β that is never corrected. The last four rows are the results when β has a known probability distribution that is updated according to the results of the inspections.

When the incorrect value of parameter β is used, the errors are significantly large. On the other hand, when Bayesian updating is applied, the incorrect prior knowledge is revised and the errors are much smaller. The peak value of parameter β is estimated with a good accuracy in all cases.

TABLE 3
STATISTICAL ANALYSIS RESULTS OF SEVERAL PRIOR DENSITY FUNCTIONS
(UNCERTAIN PARAMETER β, 2000 TRIALS)

Conditions		Number of inspections		Peak value of estimated β	
Approach	Prior knowledge	Average	Error	Average	Error
Non Bayesian	β = 40,000 (True value)	12.98	---	---	---
	β = 36,000	16.35	26.0%	---	---
	β = 44,000	10.66	-17.9%	---	---
Bayesian	Normal dist. μ = 36,000 σ = 10,000	14.59	12.4%	40060	0.2%
	Normal dist. μ = 40,000 σ = 10,000	13.56	4.5%	40460	1.2%
	Normal dist. μ = 44,000 σ = 10,000	12.52	-3.5%	41180	3.0%
	Uniform distribution	13.60	4.8%	41080	2.7%

The statistical results of several prior density functions of c are compared in Table 4. When the incorrect value of parameter c is used, the errors are relatively large. When Bayesian updating is applied, the errors decrease significantly. The peak value of parameter c is overestimated in all cases because the Bayesian estimation reaches a local minimum. Consequently, the number of inspections is underestimated. However, applying the Bayesian method improves the estimations even when prior knowledge is incorrect.

TABLE 4
STATISTICAL ANALYSIS RESULTS OF SEVERAL PRIOR DENSITY FUNCTIONS (UNCERTAIN PARAMETER c, 2000 TRIALS)

Conditions		Number of inspections		Peak value of estimated c	
Approach	Prior knowledge	Average	Error	Average	Error
Non Bayesian	c = 1.60E-4 (True value)	12.98	---	---	---
	c = 1.4E-4	14.96	14.9%	---	---
	c = 1.8E-4	11.4	-12.2%	---	---
Bayesian	Normal dist. μ = 1.4E-4 σ = 5.0E-5	12.57	-3.2%	1.77E-04	10.6%
	Normal dist. μ = 1.6E-4 σ = 5.0E-5	12.04	-7.2%	1.83E-04	14.2%
	Normal dist. μ = 1.8E-4 σ = 5.0E-5	11.43	-11.9%	1.90E-04	18.4%
	Uniform distribution of c	11.66	-10.2%	1.96E-04	22.8%

C. System Reliability

The reliability of the system is estimated as well as the accuracy of this estimated reliability.

1) Estimation of reliability

The reliability is estimated from the inspection results through a probabilistic method. Because the estimation is performed through a simulation, one can compute the true reliability of the system from the initiation information of the fatigue cracks only. Fig. 4 shows an example of the comparison of estimated and true reliabilities of the entire system, obtained through simulation with all parameters fixed. While the estimated reliability is always greater than 0.8, the true reliability reaches values as low as 0.7.

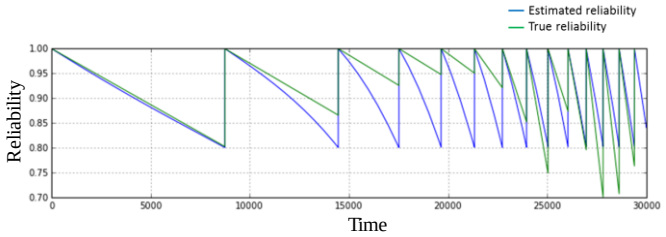


Fig. 4. Comparison of the estimated and true reliabilities (fixed parameters).

2) Accuracy of estimated reliability

The accuracy of the estimated reliability, *Aor*, is defined by (37).

$$Aor = \begin{cases} 1 & \text{when } R_{\text{true}} \geq R_{\text{design}} 0.8 \\ R_{\text{true}}/R_{\text{design}} & \text{when } R_{\text{true}} < R_{\text{design}} 0.8 \end{cases} \quad (37)$$

The three following simulations are performed: (a) simulation with fixed parameters, (b) simulation with Bayesian estimation of parameter β, (c) simulation with Bayesian estimation of parameters β and c. The value of *Aor* is shown in Table 5 and Fig. 5. The error bars in the figure correspond to the standard deviation. Using fixed parameters or performing a Bayesian estimation for uncertain parameters has no significant influence on *Aor*. These results show that by applying the Bayesian approach, the presence of uncertain parameters has no significant influence on the accuracy of the reliability estimation.

TABLE 5
STATISTICAL ANALYSIS RESULTS OF ESTIMATED RELIABILITY

Conditions	Average of *Aor*			Minimum of *Aor*		
	Mean value	SD	COV(%)	Mean value	SD	COV(%)
True values	0.925	0.052	5.6	0.656	0.185	28.2
Bayesian β	0.928	0.038	4.1	0.617	0.161	26.1
Bayesian β,c	0.904	0.057	6.3	0.610	0.172	28.2

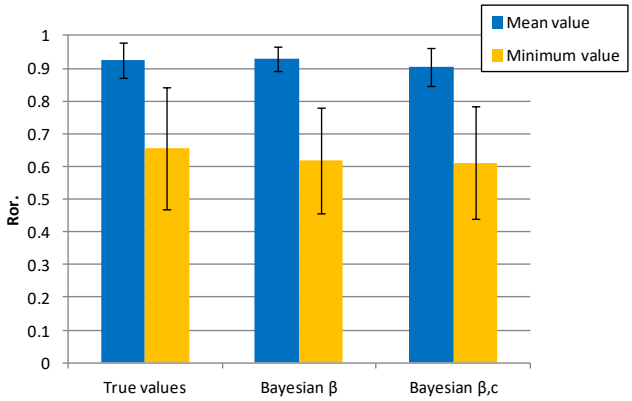


Fig. 5. Comparison of the accuracy of estimated reliability *Aor*.

VII. CONCLUSIONS

A method for determining the appropriate non-periodic inspection intervals of aircraft structures and estimating the value of uncertain parameters was improved by using conditional probabilities for the Bayesian estimation. Only the information about the last replacement or repair of a component is now required, instead of all the inspections information from start-up.

Numerical simulations showed that reasonable inspection intervals were obtained with this method, even when some of the parameters are uncertain. Furthermore, through a statistical analysis, it was shown that even with incorrect prior knowledge, the Bayesian approach allows estimating the inspection intervals with good accuracy. Finally, the accuracy of the estimated reliability of the system was evaluated statistically and it was shown that if the Bayesian approach is applied, the presence of uncertain parameters does not significantly affect the accuracy of the estimation of the reliability.

The functional equations used for fatigue crack initiation, crack propagation, probability of detection, failure rate and probability of safety are selected mainly to demonstrate the capabilities of our approach. It is a straightforward task to change any equations as well as parameters depending on the application.

ACKNOWLEDGMENT

This research was supported by an Asian Office of Aerospace R&D grant to N.P.O. Innovation of Japan (IOJ).

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